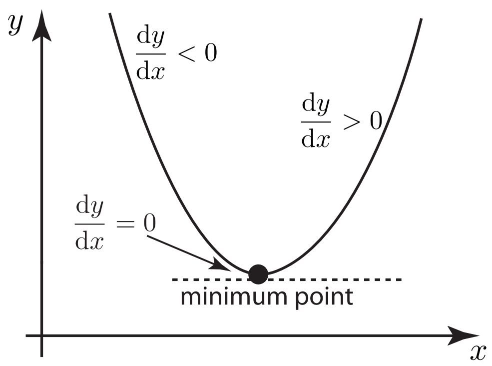
Differentiation

Differentiating a function, $y = f(x)$, we obtain its derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$. This new function tells us the gradient (slope) of the original function at any point. When $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ the gradient is zero.

$y = f(x)$	$\frac{\mathrm{d}y}{\mathrm{d}x} = f'(x)$
k, constant	0
x	1
x^2	$2x$
x^n, constant n	nx^{n-1}
$\sin(kx)$	$k\cos(kx)$
$\cos(kx)$	$-k\sin(kx)$
e^x	e^x
e^{kx}	$k\mathrm{e}^{kx}$
$\ln kx = \log_{\mathrm{e}} kx$	$\frac{1}{x}$



The linearity rules:

$$\frac{\mathrm{d}}{\mathrm{d}x}(u(x) \pm v(x)) = \frac{\mathrm{d}u}{\mathrm{d}x} \pm \frac{\mathrm{d}v}{\mathrm{d}x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(k \times f(x)) = k \times \frac{\mathrm{d}f}{\mathrm{d}x}$$

for k constant.

The product and quotient rules:

$$\frac{\mathrm{d}}{\mathrm{d}x}(uv) = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x} \qquad \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{u}{v}\right) = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}.$$

The chain rule: If $y = y(u)$ where $u = u(x)$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$.

Higher derivatives:

$f''(x)$, or $\dfrac{\mathrm{d}^2 f}{\mathrm{d}x^2}$, means differentiate $\dfrac{\mathrm{d}f}{\mathrm{d}x}$ with respect to x.

That is, $\dfrac{\mathrm{d}^2 f}{\mathrm{d}x^2} = \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\mathrm{d}f}{\mathrm{d}x}\right)$.

Partial derivatives: If $f = f(x, y)$ is a function of two (or more) independent variables, $\frac{\partial f}{\partial x}$ means differentiate f with respect to x treating y as if it were a constant. $\frac{\partial f}{\partial y}$ means differentiate f with respect to y treating x as if it were a constant.

Integration

$f(x)$	$\int f(x)\,\mathrm{d}x$
k, constant	$kx + c$
x	$\frac{x^2}{2} + c$
x^2	$\frac{x^3}{3} + c$
x^n, $(n \neq -1)$	$\frac{x^{n+1}}{n+1} + c$
$x^{-1} = \frac{1}{x}$	$\ln x + c$ or $\ln c'x$
e^x	$\mathrm{e}^x + c$
e^{kx}	$\frac{\mathrm{e}^{kx}}{k} + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
$\cos kx$	$\frac{1}{k}\sin kx + c$

The linearity rule:

$$\int (af(x) + bg(x))\,\mathrm{d}x = a\int f(x)\,\mathrm{d}x \,+\, b\int g(x)\,\mathrm{d}x, \qquad (a, b\,\mathrm{constant})$$

Integration by parts: $\displaystyle\int_a^b u\frac{\mathrm{d}v}{\mathrm{d}x}\mathrm{d}x = [uv]_a^b - \int_a^b \frac{\mathrm{d}u}{\mathrm{d}x}v\,\mathrm{d}x.$