Arithmetic and Geometric Progressions

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Introduction

Arithmetic and geometric progressions are particular types of sequences of numbers which occur frequently in business calculations. This leaflet explains these terms and shows how the sums of these sequences can be found.

Arithmetic progressions

An **arithmetic progression** is a sequence of numbers where each new term after the first is formed by adding a fixed amount called the **common difference** to the previous term in the sequence. For example the sequence

$$3, 5, 7, 9, 11 \dots$$

is an arithmetic progression. Note that having chosen the first term to be 3, each new term is found by adding 2 to the previous term, so the common difference is 2.

The common difference can be negative: for example the sequence

$$2, -1, -4, -7, \dots$$

is an arithmetic progression with first term 2 and common difference -3.

In general we can write an arithmetic progression as follows:

arithmetic progression: $a,\ a+d,\ a+2d,\ a+3d,\ \dots$

where the first term is a and the common difference is d. Some important results concerning arithmetic progressions (a.p.) now follow:

The nth term of an a.p. is given by: $a + (n-1)d$

The sum of the first n terms of an a.p. is $S_n = \dfrac{n}{2}\left(2a + (n-1)d\right)$

The sum of the terms of an arithmetic progression is known as an **arithmetic series**

Geometric progressions

A **geometric progression** is a sequence of numbers where each term after the first is found by multiplying the previous term by a fixed number called the **common ratio.** The sequence

$$1, 3, 9, 27, \ldots$$

is a geometric progression with first term 1 and common ratio 3. The common ratio could be a fraction and it might be negative. For example, the geometric progression with first term 2 and common ratio $-\frac{1}{3}$ is

$$2, -\frac{2}{3}, \frac{2}{9}, -\frac{2}{27}, \ldots$$

In general we can write a geometric progression as follows:

geometric progression: $a,\ ar,\ ar^2,\ ar^3,\ \ldots$

where the first term is a and the common ratio is r.

Some important results concerning geometric progressions (g.p.) now follow:

The nth term of a g.p. is given by: $ar^{(n-1)}$

The sum of the first n terms of a g.p. is $S_n = \dfrac{a(1-r^n)}{1-r}$ (valid only if $r \neq 1$)

The sum of the terms of a geometric progression is known as a **geometric series**.

If the common ratio in a geometric series is less than 1 in modulus, (that is $-1 < r < 1$), the sum of an infinite number of terms can be found. This is known as the **sum to infinity**, S_∞.

$S_\infty = \dfrac{a}{1-r}$ provided $-1 < r < 1$

Exercises

1. Find the 23rd term of an a.p. with first term 2 and common difference 7.
2. A g.p. is given by $1, \frac{1}{2}, \frac{1}{4}, \ldots$ What is its common ratio ?
3. Find the 7th term of a g.p. with first term 2 and common ratio 3.
4. Find the sum of the first five terms of the a.p. with first term 3 and common difference 5.
5. Find the sum of the first five terms of the g.p. with first term 3 and common ratio 2.
6. Find the sum of the infinite geometric series with first term 2 and common ratio $\frac{1}{2}$.

Answers

1. 156, 2. $\frac{1}{2}$, 3. 1458, 4. 65, 5. 93, 6. 4.

