mathcentre community project

encouraging academics to share maths support resources

mcccp-richard-4

For the help you need to support your course

Exponential and Logarithm for Economics and Business Studies

This leaflet is an overview of the properties of the functions e and $\ln$ and their applications in Economics.

Author: Morgiane Richard, University of Aberdeen

Reviewers: Anthony Cronin (University College Dublin), Shazia Ahmed (University of Glasgow)

Exponential

The exponential function is $f(t) = b^t$, where $b > 1$ is called the base.

The most commonly occurring base in Business and Economics is $e \approx 2.72$ and the corresponding exponential function is the natural exponential function $f(t) = e^t = \exp(t)$.

The number e is defined as $e = \lim_{m\to\infty}\left(1 + \frac{1}{m}\right)^m$.

Properties of e:

$(e^t)^u = (e^u)^t = e^{ut}$

$e^{t+u} = e^t e^u$

$e^{-t} = \frac{1}{e^t}$ and $e^{t-u} = \frac{e^t}{e^u}$

$\frac{d(e^t)}{dt} = e^t$ and $\frac{d(e^{f(t)})}{dt} = f'(t)e^{f(t)}$

Exponential functions in Economics:

Interest compounding: for an interest rate r compounded at frequency m on an initial principal A, the value of the asset at time t is $V(m,t) = A\left(1 + \frac{r}{m}\right)^{mt}$. In the limit $m \to \infty$ we have: $V(t) = Ae^{rt}$. The rate r can take negative values in the case of deflation or depreciation.

Rate of growth: for a function $f(t)$, the rate of growth is defined as $\frac{1}{f(t)}\frac{df}{dt}$. In the case $f(t)$ represents an exponential growth and takes the form Ae^{rt} then the rate of growth is $\frac{Are^{rt}}{Ae^{rt}} = r$.

The Cobb-Douglas Production functions: are widely common in Economics and are a family of functions taking the form: $Q = AK^\alpha L^\beta$.

Logarithm

The logarithm function log in base b is the inverse function of the exponential function in base b:

$$y = \log_b x \Leftrightarrow x = b^y$$

The natural logarithm, $\ln$ is the inverse function of the natural exponential function:

$$y = \ln x \Leftrightarrow x = e^y$$

This means that $e^{\ln x} = \ln e^x = x$ and for any base b, $b^{\log_b x} = \log_b b^x = x$.

Properties of $\ln$:

$\ln(ut) = \ln u + \ln t$

$\ln\left(\frac{1}{t}\right) = -\ln t$ and $\ln\left(\frac{t}{u}\right) = \ln t - \ln u$

$\ln(t^u) = u \ln t$

$\log_b t = \log_c t \log_b c$ and $\log_b t = \ln t \log_b e$ (conversion of base)

$\frac{d\ln(t)}{dt} = \frac{1}{t}$ and $\frac{d\ln(f(t))}{dt} = \frac{f'(t)}{f(t)} = \frac{1}{f(t)}\frac{df(t)}{dt}$

Economics applications of $\ln$

Alternative definition of rate of growth: since the rate of growth is $\frac{1}{f(t)}\frac{df}{dt} = \frac{d\ln(f(t))}{dt}$, it can also be expressed as $\frac{d\ln(f(t))}{dt}$.

Elasticity: the elasticity of a function $y(x)$ with respect to x is $\frac{d\ln y}{d\ln x} = \frac{x}{y}\frac{dy}{dx}$. If y is an exponential function of x, then the elasticity is the slope of the straight line obtained when plotting y as a function of x on a log-log graph (which is the same as plotting $\ln y$ as a function of $\ln x$).

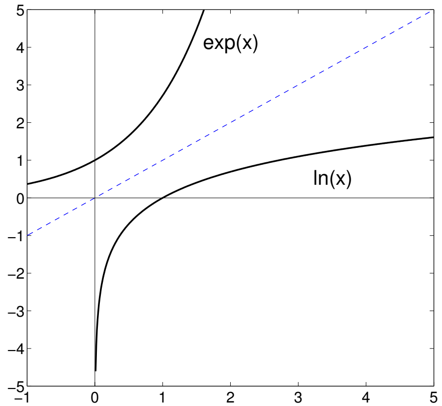

Figure 1: Graph of the functions e^x and $\ln x$.