

## Equations of Motion

mccc-Simpson-002

The *equations of motion* apply to any object moving with constant acceleration. The five equations are also known as SUVAT equations, because they each contain four of the five letters  $s$ ,  $u$ ,  $v$ ,  $a$ ,  $t$ . Here  $s$  represents displacement, the  $u$  initial velocity,  $v$  final velocity,  $a$  acceleration and  $t$  time. Given any three of these values the other two can be found using these equations. These equations are

$$\begin{array}{llll} v = u + at & (1) & v^2 = u^2 + 2as & (3) \\ s = ut + \frac{1}{2}at^2 & (2) & s = vt - \frac{1}{2}at^2 & (4) \end{array} \quad s = \frac{1}{2}(u + v)t \quad (5)$$

Equation (1) comes intuitively: if an object has a constant acceleration for a period of time, the velocity gained plus the initial velocity will be the final velocity. From this and equation (2), which is derived using the knowledge from calculus that displacement is the integral of velocity, the other equations can be derived.

For example,  $v = u + at$  can be rearranged to  $a = (v - u)/t$ . When substituted into equation (2) this gives  $s = ut + \frac{1}{2} \cdot \frac{v - u}{t} \cdot t^2 = ut + \frac{1}{2}(v - u)t = \left[ u + \frac{1}{2}(v - u) \right] t$  and finally  $s = \frac{1}{2}(u + v)t$  which is equation (5).

### Exercise

Derive equations (3) and (4) from equations (1) and (2).

### Example

A ball is kicked from rest along the ground at a constant acceleration, after 3s the ball is moving with a velocity of  $20\text{ms}^{-1}$ . How far has the ball travelled in this time?

**Solution** First, we need to select our equation. The question uses initial and final velocities, displacement and time so we need to use  $s = \frac{1}{2}(u + v)t$ . In this case we know that the initial velocity is zero because the ball starts from rest. Substitute the values:  $s = \frac{1}{2}(0 + 20) \times 3 = 30\text{m}$ .

### Example

A skydiver falling freely under the force of gravity has an initial velocity of  $18\text{ms}^{-1}$ . How long will it take him to fall 50m? Neglect air resistance and take the acceleration due to gravity to be  $9.8\text{ms}^{-2}$ .

**Solution** Again we start by choosing our equation: this time we need to use (2). Putting in the values we get;  $50 = 18t + \frac{1}{2} \cdot 9.8t^2$ , in this case we have chosen to use down as the positive direction, hence positive acceleration. Now we rearrange this to make  $t$  the subject in order to give  $4.9t^2 + 18t - 50 = 0$ . Using the quadratic formula we get  $t = \frac{-18 \pm \sqrt{18^2 - 4 \times 4.9 \times (-50)}}{2 \times 4.9}$ .

Because this is a quadratic we get two values of  $t$ , only one of which is true. Here there is a positive and a negative value, and given the context of the question we take the positive value as a negative time would mean the event had already happened. Thus the answer is 1.85s to 3 significant figures.



## Exercise

1] A car is moving along a straight horizontal road with a velocity of  $u$ , it accelerates uniformly at a rate of  $15\text{ms}^{-2}$  for  $25\text{m}$  giving a velocity of  $30\text{ms}^{-1}$ . What is the value of  $u$ ?

2] These equations of motion can actually be applied to real life driving as well. For example, the Highway Code states that if you are travelling at  $70\text{mph}$  (about  $31.29\text{ms}^{-1}$ ) the stopping distance will be  $75\text{m}$ . What deceleration does that assume?

## Answers

1]  $u = 12.247\text{ms}^{-1}$

2] Deceleration =  $6.528\text{ms}^{-2}$

## Motion in 2 dimensions

Up to now we have looked at objects moving in one dimension. However, the modelling of motion in a second dimension, such as the trajectory of an object moving both horizontally and vertically, is done in a similar way. In this case we consider the two separately, as one does not effect the other — an example is that no matter how hard you push down on an object it will never move sideways.

### Example

A rock is thrown horizontally off a  $100\text{ metre}$  high cliff at  $12\text{ms}^{-1}$ . How far from the base of the cliff will it land? Neglect air resistance and take the acceleration due to gravity to be  $9.8\text{ms}^{-2}$ .

**Solution** As the the horizontal and vertical motions are unrelated the only common value between the two will be the time. We want to know how far from the cliff it lands, so we are looking at the horizontal distance travelled. However, as it stands we only know the initial velocity and the acceleration of the horizontal motion, so first we must consider its vertical movement in order to find out the time taken to reach the ground. Vertically we know that the initial velocity is zero, as it was only given a horizontal impulse, we know that the height of the cliff is  $100\text{m}$  and that the acceleration due to gravity is  $9.8\text{ms}^{-2}$ , with down as the positive direction. Given what we know we must use the equation  $s = ut + \frac{1}{2}at^2$ . Substituting values gives  $100 = 0t + \frac{1}{2} \cdot 9.8t^2$ , which can be simplified to  $100 = 4.9t^2$ , hence  $t = 4.517539515$ . This value of  $t$  is the time at which the rock will reach the ground, which will be the same no matter which direction we are looking at so we can use this value of  $t$  along with the data in the question to set up an equation for the horizontal motion of the rock. We know that horizontally the initial velocity of the rock is  $12\text{ms}^{-1}$ , the acceleration is zero as its horizontal velocity is constant, and we now know that the time is  $t = 4.517539515\text{s}$ . With that we can find the displacement using  $s = ut + \frac{1}{2}at^2$  and as there is no acceleration this simplifies to  $\text{distance} = \text{speed} \times \text{time}$ . So the distance is  $12 \times 4.517539515 = 54.2\text{m}$  to 3 significant figures.

## Final points

So far in every example we have looked at the trajectory has been either horizontal or vertical. However, this will not always be the case, so sometimes you will need to use trigonometry to find the components of  $u$  in the two directions. Say for example an object is projected with an initial velocity of  $15\text{ms}^{-1}$  at an angle of  $40^\circ$  above the horizontal, the horizontal value of  $u$  would be  $15 \cos 40^\circ \text{ms}^{-1}$  and the vertical value of  $u$  would be  $15 \sin 40^\circ \text{ms}^{-1}$ , and other than that everything else would be worked out in the same way, just using those as the values of  $u$ .

It should be noted that to find the highest point of a flight, that will be the point at which the object has finished moving upwards and has not started moving back down again, so its vertical velocity will be zero. Also, as you use displacement rather than distance, if an object is being projected vertically upwards, then returns to the starting point the overall displacement would be zero..

