Engineering Maths First Aid Kit

2.1

The laws of indices

Introduction

A **power**, or an **index**, is used to write a product of numbers very compactly. The plural of index is **indices**. In this leaflet we remind you of how this is done, and state a number of rules, or laws, which can be used to simplify expressions involving indices.

1. Powers, or indices

We write the expression

$$3 \times 3 \times 3 \times 3 \quad \text{as} \quad 3^4$$

We read this as 'three to the power four'.

Similarly

$$z \times z \times z = z^3$$

We read this as 'z to the power three' or 'z cubed'.

In the expression b^c, the **index** is c and the number b is called the **base**. Your calculator will probably have a button to evaluate powers of numbers. It may be marked x^y. Check this, and then use your calculator to verify that

$$7^4 = 2401 \qquad \text{and} \qquad 25^5 = 9765625$$

Exercises

1. Without using a calculator work out the value of

a) 4^2, b) 5^3, c) 2^5, d) $\left(\frac{1}{2}\right)^2$, e) $\left(\frac{1}{3}\right)^2$, f) $\left(\frac{2}{5}\right)^3$.

2. Write the following expressions more concisely by using an index.

a) $a \times a \times a \times a$, b) $(yz) \times (yz) \times (yz)$, c) $\left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right)$.

Answers

1. a) 16, b) 125, c) 32, d) $\frac{1}{4}$, e) $\frac{1}{9}$, f) $\frac{8}{125}$.

2. a) a^4, b) $(yz)^3$, c) $\left(\frac{a}{b}\right)^3$.

2. The laws of indices

To manipulate expressions involving indices we use rules known as the **laws of indices**. The laws should be used precisely as they are stated - do not be tempted to make up variations of your own! The three most important laws are given here:

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First law

$$a^m \times a^n = a^{m+n}$$

When expressions with the same base are multiplied, the indices are added.

Example

We can write

$$7^6 \times 7^4 = 7^{6+4} = 7^{10}$$

You could verify this by evaluating both sides separately.

Example

$$z^4 \times z^3 = z^{4+3} = z^7$$

Second Law

$$\frac{a^m}{a^n} = a^{m-n}$$

When expressions with the same base are divided, the indices are subtracted.

Example

We can write

$$\frac{8^5}{8^3} = 8^{5-3} = 8^2 \qquad \text{and similarly} \qquad \frac{z^7}{z^4} = z^{7-4} = z^3$$

Third law

$$(a^m)^n = a^{mn}$$

Note that m and n have been multiplied to yield the new index mn.

Example

$$(6^4)^2 = 6^{4\times 2} = 6^8 \qquad \text{and} \qquad (e^x)^y = e^{xy}$$

It will also be useful to note the following important results:

$$a^0 = 1, \qquad a^1 = a$$

Exercises

1. In each case choose an appropriate law to simplify the expression:

a) $5^3 \times 5^{13}$, b) $8^{13} \div 8^5$, c) $x^6 \times x^5$, d) $(a^3)^4$, e) $\frac{y^7}{y^3}$, f) $\frac{x^8}{x^7}$.

2. Use one of the laws to simplify, if possible, $a^6 \times b^5$.

Answers

1. a) 5^{16}, b) 8^8, c) x^{11}, d) a^{12}, e) y^4, f) $x^1 = x$.

2. This cannot be simplified because the bases are not the same.

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