Engineering Maths First Aid Kit

2.6

Factorising quadratics

Introduction

On this leaflet we explain the procedure for factorising quadratic expressions such as x^2+5x+6.

1. Factorising quadratics

You will find that you are expected to be able to factorise expressions such as x^2+5x+6.

First of all note that by removing the brackets from

$$(x+2)(x+3)$$

we find

$$(x+2)(x+3) = x^2+2x+3x+6 = x^2+5x+6$$

When we **factorise** x^2+5x+6 we are looking for the answer $(x+2)(x+3)$.

It is often convenient to do this by a process of educated guesswork and trial and error.

Example
Factorise x^2+6x+5.

Solution
We would like to write x^2+6x+5 in the form

$$(\quad + \quad)(\quad + \quad)$$

First note that we can achieve the x^2 term by placing an x in each bracket:

$$(x \quad + \quad)(x \quad + \quad)$$

The next place to look is the constant term in x^2+6x+5, that is, 5. By removing the brackets you will see that this is calculated by multiplying the two numbers in the brackets together. We seek two numbers which multiply together to give 5. Clearly 5 and 1 have this property, although there are others. So

$$x^2+6x+5 = (x+5)(x+1)$$

At this stage you should always remove the brackets again to check.

The factors of x^2+6x+5 are $(x+5)$ and $(x+1)$.

Example
Factorise x^2-6x+5.

Solution
Again we try to write the expression in the form

$$x^2 - 6x + 5 = (x \quad + \qquad)(x \quad + \qquad)$$

And again we seek two numbers which multiply to give 5. However this time 5 and 1 will not do, because using these we would obtain a middle term of $+6x$ as we saw in the last example. Trying -5 and -1 will do the trick.

$$x^2 - 6x + 5 = (x - 5)(x - 1)$$

You see that some thought and perhaps a little experimentation is required.

You will need even more thought and care if the coefficient of x^2, that is the number in front of the x^2, is anything other than 1. Consider the following example.

Example
Factorise $2x^2 + 11x + 12$.

Solution
Always start by trying to obtain the correct x^2 term:

We write

$$2x^2 + 11x + 12 = (2x \quad + \qquad)(x \quad + \qquad)$$

Then study the constant term 12. It has a number of pairs of factors, for example 3 and 4, 6 and 2 and so on. By trial and error you will find that the correct factorisation is

$$2x^2 + 11x + 12 = (2x + 3)(x + 4)$$

but you will only realise this by removing the brackets again.

Exercises
1. Factorise each of the following:

a) x^2+5x+4, b) x^2-5x+4, c) x^2+3x-4, d) x^2-3x-4, e) $2x^2-13x-7$, f) $2x^2+13x-7$, g) $3x^2-2x-1$, h) $3x^2+2x-1$, i) $6x^2+13x+6$.

Answers
1. a) $(x+1)(x+4)$, b) $(x-1)(x-4)$, c) $(x-1)(x+4)$, d) $(x+1)(x-4)$, e) $(2x+1)(x-7)$, f) $(2x-1)(x+7)$, g) $(3x+1)(x-1)$, h) $(3x-1)(x+1)$, i) $(3x+2)(2x+3)$.