The exponential constant e

Introduction

The letter e is used in many mathematical calculations to stand for a particular number known as the **exponential constant**. This leaflet provides information about this important constant, and the related **exponential function**.

1. The exponential constant

The exponential constant is an important mathematical constant and is given the symbol e. Its value is approximately 2.718. It has been found that this value occurs so frequently when mathematics is used to model physical and chemical phenomena that it is convenient to write simply e.

It is often necessary to work out powers of this constant, such as e^2, e^3 and so on. Your scientific calculator will be programmed to do this already. You should check that you can use your calculator to do this. Look for a button marked e^x, and check that

$$e^2 = 7.389, \qquad \text{and} \qquad e^3 = 20.086$$

In both cases we have quoted the answer to three decimal places although your calculator will give a more accurate answer than this.

You should also check that you can evaluate negative and fractional powers of e such as

$$e^{1/2} = 1.649 \qquad \text{and} \qquad e^{-2} = 0.135$$

2. The exponential function

If we write $y = e^x$ we can calculate the value of y as we vary x. Values obtained in this way can be placed in a table. For example:

x	-3	-2	-1	0	1	2	3
$y = e^x$	0.050	0.135	0.368	1	2.718	7.389	20.086

This is a table of values of the **exponential function** e^x. If pairs of x and y values are plotted we obtain a **graph** of the exponential function as shown overleaf. If you have never seen this

function before it will be a worthwhile exercise to plot it for yourself.
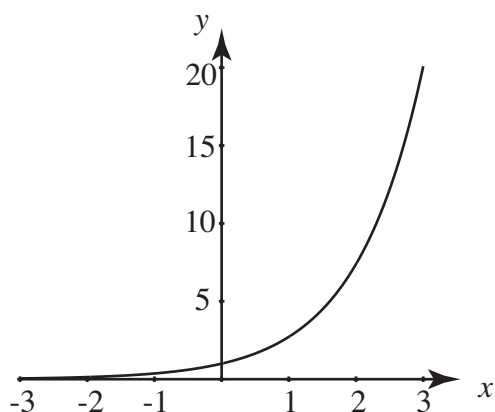
A graph of the exponential function $y = e^x$

It is important to note that as x becomes larger, the value of e^x grows without bound. We write this mathematically as $e^x \to \infty$ as $x \to \infty$. This behaviour is known as **exponential growth**.

3. The negative exponential function

A related function is the **negative exponential function** $y = e^{-x}$. A table of values of this function is shown below together with its graph.

x	-3	-2	-1	0	1	2	3
$y = e^{-x}$	20.086	7.389	2.718	1	0.368	0.135	0.050
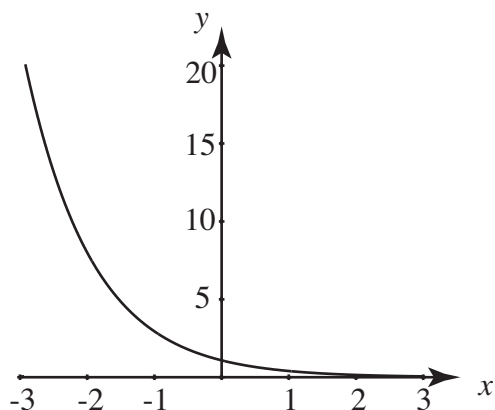
A graph of the negative exponential function $y = e^{-x}$

It is very important to note that as x becomes larger, the value of e^{-x} approaches zero. We write this mathematically as $e^{-x} \to 0$ as $x \to \infty$. This behaviour is known as **exponential decay**.

Exercises

A useful exercise would be to draw up tables of values and plot graphs of some related functions:

a) $y = e^{2x}$,　　　b) $y = e^{0.5x}$,　　　c) $y = -e^x$,　　　d) $y = -e^{-x}$,　　　e) $y = 1 - e^{-x}$.