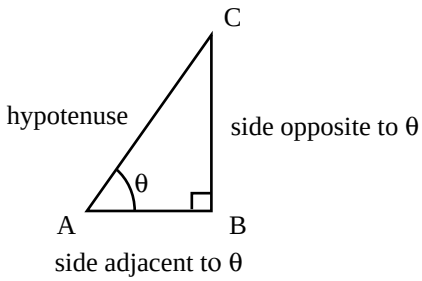
The Trigonometrical Ratios

Introduction

The trigonometrical ratios sine, cosine and tangent appear frequently in many engineering problems. This leaflet revises the meaning of these terms.

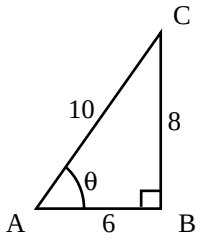
1. Sine, cosine and tangent ratios

Study the right-angled triangle ABC shown below.



The side opposite the right-angle is called the **hypotenuse**. The side **opposite** to θ is BC. The remaining side, AB, is said to be **adjacent** to θ.

Suppose we know the lengths of each of the sides as in the figure below.



We can then divide the length of one side by the length of one of the other sides.

The ratio $\frac{BC}{AC}$ is known as the **sine** of angle θ. This is abbreviated to $\sin\theta$. In the triangle shown we see that

$$\sin\theta = \frac{8}{10} = 0.8$$

The ratio $\frac{AB}{AC}$ is known as the **cosine** of angle θ. This is abbreviated to $\cos\theta$. In the triangle shown we see that

$$\cos\theta = \frac{6}{10} = 0.6$$

The ratio $\frac{BC}{AB}$ is known as the **tangent** of angle θ. This is abbreviated to $\tan\theta$. In the triangle shown we see that

$$\tan\theta = \frac{8}{6} = 1.3333$$

In any right-angled triangle we define the trigonometrical ratios as follows:

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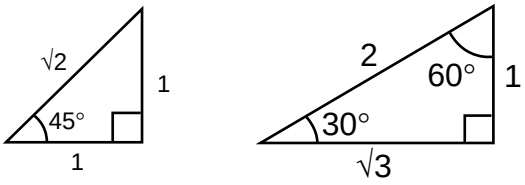
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AC} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AB}$$

2. Some standard, or common, triangles



$$\sin 45^\circ = \frac{1}{\sqrt{2}}, \qquad \cos 45^\circ = \frac{1}{\sqrt{2}}, \qquad \tan 45^\circ = 1$$

$$\sin 30^\circ = \frac{1}{2}, \qquad \cos 30^\circ = \frac{\sqrt{3}}{2}, \qquad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \qquad \cos 60^\circ = \frac{1}{2}, \qquad \tan 60^\circ = \sqrt{3}$$

3. Using a calculator

If we know the angles in a right-angled triangle the trigonometrical ratios can be found using a scientific calculator. Look for the sine, cosine and tangent buttons on your calculator and make sure that you can use them by verifying that

$$\sin 50^\circ = 0.7660, \qquad \cos 32^\circ = 0.8480$$

Your calculator will be able to handle angles measured in either radians or degrees. It will be necessary for you to choose the appropriate units. Study your calculator manual to learn how to do this. Check that

$$\sin 0.56 \text{ radians} = 0.5312, \qquad \tan 1.4 \text{ radians} = 5.7979$$

4. Finding an angle when a trigonometrical ratio is known

If we are given, or know, a value for $\sin\theta$, $\cos\theta$ or $\tan\theta$ we may want to work out the corresponding angle θ. This process is known as finding the inverse sine, inverse cosine or inverse tangent. Your calculator will be pre-programmed for doing this. The buttons will be labelled invsin, or $\sin^{-1}$, and so on.

Check that you can use your calculator to show that if $\sin\theta = 0.75$ then $\theta = 48.59^\circ$.

Mathematically we write this as follows:

$$\text{if } \sin\theta = 0.75, \quad \text{then } \quad \theta = \sin^{-1} 0.75 = 48.59^\circ$$

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