Linear functions

In mathematics and statistics a **function** is used to relate one variable to another variable. The simplest type of function is known as a **linear function**. This leaflet explains linear functions and the conventions we use when describing them. This leaflet should be read in conjunction with the leaflet $x - y$ *plots*.

Dependent and independent variables

Suppose we consider the relationship between the heights of a group of school children and their ages. It is reasonable to assume that, in general, the height will change as the age changes - we say that the height **depends upon** the age. Furthermore, it may be reasonable to assume that as the age of a child increases, so too will his or her height. Height and age are the two **variables** being considered. Because the height *depends upon* the age we say that height is the **dependent variable** and we refer to the age as the **independent variable**.

As a second example from epidemiology, we might be interested in the number of new cases of a disease diagnosed each day over a three month time period. Clearly, the number of new cases is likely to change as time goes on. In this example the two variables are the number of new cases, and time. The dependent variable is the number of the new cases, and the independent variable is time.
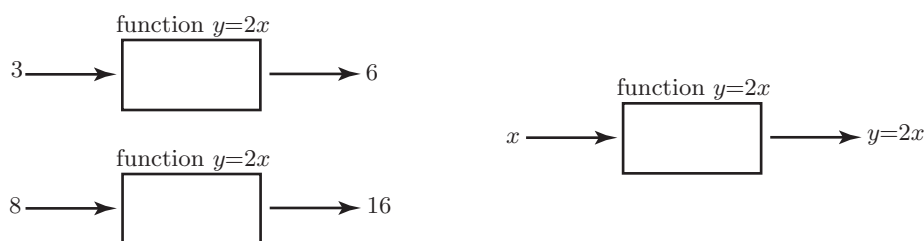
Functions

The relationship between a dependent and an independent variable can often be expressed mathematically using a formula. In this context we refer to this formula as a **function**. The function is used to calculate a value for the dependent variable when we choose a value for the independent variable.

For example, consider the function $y = 2x$. By choosing a value for x we can use this to calculate a value for y. Thus x is the independent variable and y is the dependent variable.

So, if we choose $x = 3$, then $y = 2 \times 3 = 6$.

If we choose $x = 8$, then $y = 2 \times 8 = 16$.

We can think of the function as a mathematical machine into which we put the value we have chosen for x, and out of which comes the calculated value of y. This idea is illustrated in the diagrams below.

The input to the function is always the independent variable; the output is always the dependent variable, because the value of the output *depends upon* the value of the input.

Linear functions

Linear functions are those which have the special form $y = a + bx$ where a and b are constants, that is numbers. These constants can be positive, negative and zero. So, the following are all examples of linear functions

$$
\begin{aligned}
y &= 4 + 9x &&\text{here } a = 4 \text{ and } b = 9 \\
y &= 3 - 2x &&\text{here } a = 3 \text{ and } b = -2 \\
y &= 8x &&\text{here } a = 0 \text{ and } b = 8 \\
y &= 9 &&\text{here } a = 9 \text{ and } b = 0 \\
y &= -1 - x &&\text{here } a = -1 \text{ and } b = -1
\end{aligned}
$$

Linear functions have the form $y = a + bx$ where a and b are constants (numbers).

The graph of a linear function

Linear functions are so-called because if you plot the points with coordinates (x, y) we find that all points lie on a **straight line**. The line can slope upwards, downwards, and in special cases may be horizontal or vertical.

Example

Suppose we wish to plot a graph of the linear function $y = 3 + 2x$. We choose a range of x values, say $x = -3$ to $x = 3$. We use the function $y = 3 + 2x$ to calculate corresponding values for y.

For example, when $x = -3$, $y = 3 + 2(-3) = 3 - 6 = -3$, giving rise to the coordinate pair $(-3, -3)$.

Doing this for several points we can produce the table below.

x	-3	-2	-1	0	1	2	3
y	-3	-1	1	3	5	7	9

Each point (x, y) is then plotted and the points are joined to produce the graph shown. Notice that it is a straight line. When plotting a straight line graph it is only necessary to plot two points and join them. However it is advisable to plot a third to ensure that you have calculated correctly.

Note that the independent variable is always plotted on the horizontal axis; the dependent variable is always plotted on the vertical axis.
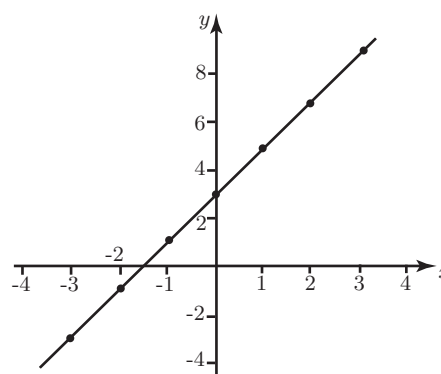
The leaflet *The equation and graph of a straight line* extends the ideas presented here and discusses how we describe the slope of a straight line graph.

Exercises

1. Evaluate the linear function $y = 3x + 7$ when x is 4, -3, 0, and 14. Sketch its graph.

Answers

1. 19, -2, 7, 49.