

# Quadratic equations 1

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## Introduction

This leaflet will explain how many quadratic equations can be solved by **factorisation**.

## Quadratic equations

A **quadratic equation** is an equation of the form  $ax^2 + bx + c = 0$ , where  $a$ ,  $b$  and  $c$  are constants. For example  $3x^2 + 2x - 9 = 0$  is a quadratic equation with  $a = 3$ ,  $b = 2$  and  $c = -9$ .

The constants  $b$  and  $c$  can have any value including 0. The constant  $a$  can have any value except 0. This is to ensure that the equation has an  $x^2$  term. We often refer to  $a$  as the coefficient of  $x^2$ , to  $b$  as the coefficient of  $x$  and to  $c$  as the constant term. Usually,  $a$ ,  $b$  and  $c$  are known numbers, whilst  $x$  represents an unknown quantity which we will be trying to find.

## The solutions of a quadratic equation

To **solve** a quadratic equation we must find values for  $x$  which when substituted into the equation make the left-hand and right-hand sides equal. These values are also called **roots**. For example the value  $x = 4$  is a solution of the equation  $x^2 - 3x - 4 = 0$  because substituting 4 for  $x$  we find

$$4^2 - 3(4) - 4 = 16 - 12 - 4$$

which simplifies to zero, the same as the right-hand side of the equation. There are several techniques which can be used to solve quadratic equations. One of these, *factorisation*, is discussed in this leaflet. You should be aware that not all quadratic equations can be solved by this method. An alternative method which uses a formula is described on leaflet *Quadratic Equations 2*.

## Solving a quadratic equation by factorisation.

Sometimes, but not always, it is possible to solve a quadratic equation using factorisation. If you need to revise factorisation you should see leaflet *Factorising quadratics*.

## Example

Solve the equation  $x^2 + 7x + 12 = 0$  by factorisation.

## Solution

We first factorise  $x^2 + 7x + 12$  as  $(x + 3)(x + 4)$ . Then the equation becomes  $(x + 3)(x + 4) = 0$ .

It is important that you realise that if the product of two quantities is zero, then one or both of the quantities must be zero. It follows that either

$$x + 3 = 0, \text{ that is } x = -3 \quad \text{or} \quad x + 4 = 0, \text{ that is } x = -4$$

The roots of  $x^2 + 7x + 12 = 0$  are  $x = -3$  and  $x = -4$ .

### Example

Solve the quadratic equation  $x^2 + 4x - 21 = 0$ .

### Solution

$x^2 + 4x - 21$  can be factorised as  $(x + 7)(x - 3)$ . Then

$$\begin{aligned}x^2 + 4x - 21 &= 0 \\(x + 7)(x - 3) &= 0\end{aligned}$$

Then either

$$x + 7 = 0, \text{ that is } x = -7 \quad \text{or} \quad x - 3 = 0, \text{ that is } x = 3$$

The roots of  $x^2 + 4x - 21 = 0$  are  $x = -7$  and  $x = 3$ .

### Example

Find the roots of the quadratic equation  $x^2 - 10x + 25 = 0$ .

### Solution

$$x^2 - 10x + 25 = (x - 5)(x - 5) = (x - 5)^2$$

Then

$$\begin{aligned}x^2 - 10x + 25 &= 0 \\(x - 5)^2 &= 0 \\x &= 5\end{aligned}$$

There is one root,  $x = 5$ . Such a root is called a **repeated root**.

### Example

Solve the quadratic equation  $2x^2 + 3x - 2 = 0$ .

### Solution

The equation is factorised to give  $(2x - 1)(x + 2) = 0$ . So, from  $2x - 1 = 0$  we find  $2x = 1$ , that is  $x = \frac{1}{2}$ . From  $x + 2 = 0$  we find  $x = -2$ . The two solutions are therefore  $x = \frac{1}{2}$  and  $x = -2$ .

### Exercises

1. Solve the following quadratic equations by factorization.

a)  $x^2 + 7x + 6 = 0$ ,    b)  $x^2 - 8x + 15 = 0$ ,    c)  $x^2 - 9x + 14 = 0$ ,

d)  $2x^2 - 5x - 3 = 0$ ,    e)  $6x^2 - 11x - 10 = 0$ ,    f)  $6x^2 + 13x + 6 = 0$ .

### Answers

a)  $-1, -6$ ,    b)  $3, 5$ ,    c)  $2, 7$ ,    d)  $3, -\frac{1}{2}$ ,    e)  $\frac{5}{2}, -\frac{2}{3}$ ,    f)  $x = -\frac{3}{2}, x = -\frac{2}{3}$ .