centre for excellence
sigma Σ
in mathematics & statistics support

Complex numbers

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In this unit we describe formally what is meant by a **complex number**. First let us revisit the solution of a quadratic equation.

Example Use the formula for solving a quadratic equation to solve $x^2 - 10x + 29 = 0$.

Solution Using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with $a = 1$, $b = -10$ and $c = 29$, we find

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(29)}}{2}$$

$$x = \frac{10 \pm \sqrt{100 - 116}}{2}$$

$$x = \frac{10 \pm \sqrt{-16}}{2}$$

Now using i we can find the square root of -16 as 4i, and then write down the two solutions of the equation.

$$x = \frac{10 \pm 4\,\mathrm{i}}{2} = 5 \pm 2\,\mathrm{i}$$

The solutions are $x = 5+2$i and $x = 5-2$i.

Real and imaginary parts

We have found that the solutions of the equation $x^2 - 10x + 29 = 0$ are $x = 5 \pm 2$i. The solutions are known as **complex numbers**. A complex number such as $5 + 2$i is made up of two parts, a **real part** 5, and an **imaginary part** 2. The imaginary part is the multiple of i.

It is common practice to use the letter z to stand for a complex number and write $z = a + b\,\mathrm{i}$ where a is the real part and b is the imaginary part.

Key Point

If z is a **complex number** then we write

$$z = a + b\,\mathrm{i} \qquad \text{where } \mathrm{i} = \sqrt{-1}$$

where a is the **real part** and b is the **imaginary part**.

Example

State the real and imaginary parts of $3 + 4\mathrm{i}$.

Solution

The real part is 3.

The imaginary part is 4.

Example

State the real and imaginary parts of $-2 + 5\mathrm{i}$.

Solution

The real part is -2.

The imaginary part is 5.

Example

State the real and imaginary parts of $-3 - 9\mathrm{i}$.

Solution

The real part is -3.

The imaginary part is -9.

Example

State the real and imaginary parts of $5\mathrm{i}$.

Solution

In this example, there is no real part. In other words, the real part is 0.

The imaginary part is 5. This number is purely imaginary.

Example

State the real and imaginary parts of 17.

Solution

The real part is 17.

There is no imaginary part. In other words, the imaginary part is 0. We can think of 17 as $17 + 0\mathrm{i}$.

In fact all real numbers can be thought of as complex numbers which have zero imaginary part.

In the following unit we will look at how complex numbers can be added, subtracted, multiplied and divided.