Rearranging formulas 1

Introduction

The ability to rearrange formulas or rewrite them in different ways is an important skill. This leaflet will explain how to rearrange some simple formulas.

The subject of a formula

Many business students will be familiar with the formula for simple interest which states that $I = Pin$. Here, I is the interest earned, P is the principal or amount invested, i is the interest rate per time period, and n is the number of time periods. If the values of P, i and n are known then the formula $I = Pin$ enables us to calculate the value of I. In the form $I = Pin$, we say that the **subject** of the formula is I. Usually the subject of a formula is on its own on the left-hand side. This formula can be written in other ways:

$$P = \frac{I}{in}, \qquad i = \frac{I}{Pn}, \qquad n = \frac{I}{Pi}$$

In the first case P is the subject of the formula whilst in the second case i is the subject. If we know values of I, i and n we can use $P = \frac{I}{in}$ to find P. On the other hand, if we know values of I, P and n, we can use $i = \frac{I}{Pn}$ to find i. Similar comments apply with regard to the other rearrangement. So you see, it is important to be able to write formulas in different ways, so that we can make a particular variable the subject.

Rules for rearranging, or transposing, a formula

You can think of a formula as a pair of balanced scales. The quantity on the left is equal to the quantity on the right. If we add an amount to one side of the scale pans, say the left one, then to keep balance we must add the same amount to the pan on the right. Similarly if we take away an amount from the left, we must take the same amount away from the pan on the right. The same applies to formulas. If we add an amount to one side, we must add the same to the other to keep the formula valid. If we subtract an amount from one side we must subtract the same amount from the other. Furthermore, if we multiply the left by any amount, we must multiply the right by the same amount. If we divide the left by any amount we must divide the right by the same amount. When you are trying to rearrange, or **transpose**, a formula, keep these operations clearly in mind.

To transpose or rearrange a formula you may

- add or subtract the same quantity to or from both sides
- multiply or divide both sides by the same quantity

Later, we shall see that a further group of operations is allowed, but first get some practice with these Examples and Exercises.

Example

Rearrange the formula $y = x + 8$ in order to make x the subject instead of y.

Solution

To make x the subject we must remove the 8 from the right. So, we subtract 8 from the right, but we remember that we must do the same to the left. So

$$
\begin{aligned}
\text{if} \quad y &= x + 8, \quad \text{subtracting 8 yields} \\
y - 8 &= x + 8 - 8 \\
y - 8 &= x
\end{aligned}
$$

We have x on its own, although it is on the right. This is no problem since if $y - 8$ equals x, then x equals $y - 8$, that $x = y - 8$. We have succeeded in making x the subject of the formula.

Example

Rearrange the formula $y = 3x$ to make x the subject.

Solution

The reason why x does not appear on its own is that it is multiplied by 3. If we divide $3x$ by 3 we obtain $\dfrac{3x}{3} = x$. So, we can obtain x on its own by dividing both sides of the formula by 3.

$$
\begin{aligned}
y &= 3x \\
\frac{y}{3} &= \frac{3x}{3} \\
&= x
\end{aligned}
$$

Finally $x = \dfrac{y}{3}$ and we have succeeded in making x the subject of the formula.

Example

Rearrange $y = 11 + 7x$ to make x the subject.

Solution

Starting from $y = 11 + 7x$ we subtract 11 from each side to give $y - 11 = 7x$. Then, dividing both sides by 7 gives $\dfrac{y - 11}{7} = x$. Finally $x = \dfrac{y - 11}{7}$.

Exercises

1. Transpose each of the following formulas to make x the subject.

a) $y = x - 7$,   b) $y = 2x - 7$,   c) $y = 2x + 7$,   d) $y = 7 - 2x$,   e) $y = \frac{x}{5}$.

2. Transpose each of the following formulas to make v the subject.

a) $w = 3v$,   b) $w = \frac{1}{3}v$,   c) $w = \frac{v}{3}$,   d) $w = \frac{2v}{3}$,   e) $w = \frac{2}{3}v$.

Answers

1. a) $x = y + 7$,   b) $x = \frac{y+7}{2}$,   c) $x = \frac{y-7}{2}$,   d) $x = \frac{7-y}{2}$,   e) $x = 5y$.

2. a) $v = \frac{w}{3}$,   b) $v = 3w$,   c) same as b),   d) $v = \frac{3w}{2}$,   e) same as d).