Powers and roots

Introduction

Powers are used when we want to multiply a number by itself repeatedly.

Powers

When we wish to multiply a number by itself we use **powers**, or **indices** as they are also called.

For example, the quantity $7 \times 7 \times 7 \times 7$ is usually written as 7^4. The number 4 tells us the number of sevens to be multiplied together. In this example, the power, or index, is 4. The number 7 is called the **base**.

Example

$6^2 = 6 \times 6 = 36$. We say that '6 squared is 36', or '6 to the power 2 is 36'.

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. We say that '2 to the power 5 is 32'.

Your calculator will be pre-programmed to evaluate powers. Most calculators have a button marked x^y, or alternatively $\char`\^$. Ensure that you are using your calculator correctly by verifying that $3^{11} = 177147$.

Square roots

When 5 is squared we obtain 25. That is $5^2 = 25$.

The reverse of this process is called **finding a square root**. The square root of 25 is 5. This is written as $\sqrt[2]{25} = 5$, or simply $\sqrt{25} = 5$.

Note also that when -5 is squared we again obtain 25, that is $(-5)^2 = 25$. This means that 25 has another square root, -5.

In general, a square root of a number is a number which when squared gives the original number. There are always two square roots of any positive number, one positive and one negative. However, negative numbers do not possess any square roots.

Most calculators have a square root button, probably marked $\sqrt{}$. Check that you can use your calculator correctly by verifying that $\sqrt{79} = 8.8882$, to four decimal places. Your calculator will only give the positive square root but you should be aware that the second, negative square root is -8.8882.

An important result is that the square root of a product of two numbers is equal to the product of the square roots of the two numbers. For example

$$\sqrt{16 \times 25} = \sqrt{16} \times \sqrt{25} = 4 \times 5 = 20$$

More generally,

$$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$$

However your attention is drawn to a common error. It is not true that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$. Substitute some simple values for yourself to see that this cannot be right.

Exercises

1. Without using a calculator write down the value of $\sqrt{9 \times 36}$.

2. Find the square of the following: a) $\sqrt{2}$,   b) $\sqrt{12}$.

3. Show that the square of $5\sqrt{2}$ is 50.

Answers

1. 18, (and also -18).   2. a) 2,   b) 12.

Cube roots and higher roots

The cube root of a number, is the number which when cubed gives the original number. For example, because $4^3 = 64$ we know that the cube root of 64 is 4, written $\sqrt[3]{64} = 4$. All numbers, both positive and negative, possess a single cube root.

Higher roots are defined in a similar way: because $2^5 = 32$, the fifth root of 32 is 2, written $\sqrt[5]{32} = 2$.

Exercises

1. Without using a calculator find a) $\sqrt[3]{27}$,   b) $\sqrt[3]{125}$.

Answers

1. a) 3,   b) 5.

Surds

Expressions involving roots, for example $\sqrt{2}$ and $5\sqrt[3]{2}$ are also known as **surds**. It is usually quite acceptable to leave an answer in surd form rather than calculating its decimal approximation with a calculator.

It is often possible to write surds in equivalent forms. For example, $\sqrt{48}$ can be written as $\sqrt{3 \times 16}$, that is $\sqrt{3} \times \sqrt{16} = 4\sqrt{3}$.

Exercises

1. Write the following in their simplest surd form: a) $\sqrt{180}$,   b) $\sqrt{63}$.

2. By multiplying numerator and denominator by $\sqrt{2} + 1$, show that

$$\frac{1}{\sqrt{2}-1} \quad \text{is equivalent to} \quad \sqrt{2}+1$$

Answers

1. a) $6\sqrt{5}$,   b) $3\sqrt{7}$.

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