Equilibrium of a particle

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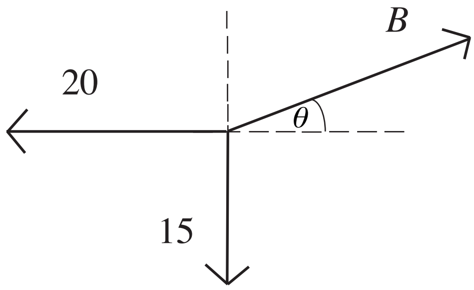
A particle is in equilibrium if the vector sum of the external forces acting on it is zero. Hence a particle is in equilibrium if:
1. It is at rest and remains at rest – Static Equilibrium
2. It moves with constant velocity – Dynamic Equilibrium

If there are only two forces acting on a particle that is in equilibrium, then the two forces must be equal (in magnitude) and opposite in direction to each other. If three forces act on a particle that is in equilibrium, then when the three forces are placed end to end they must form a triangle.

Problems involving 3 or more forces can be solved in a variety of ways, including the sine and cosine rules used in leaflet 1.5 (Force as a vector) and by resolving forces in two perpendicular directions used in leaflet 1.7 (Resolving forces, **i**, **j** notation). This second method is perhaps most versatile and hence is more commonly used.

Worked Example 1.

The three forces in the diagram are in equilibrium. What are the values of B and θ?



Solution

Resolving horizontally: $B\cos\theta - 20 = 0$ (1)
Resolving vertically: $B\sin\theta - 15 = 0$ (2)

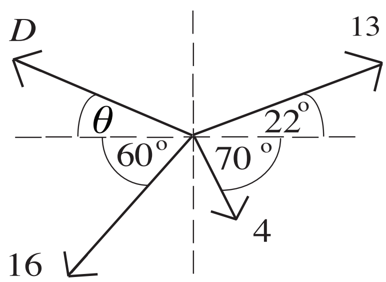
It is often the case that simultaneous equations like (1) and (2) occur in such problems. These can be solved using a variety of methods. In some cases trigonometric identities i.e. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, need to be used.

From (1): $B = \dfrac{20}{\cos\theta}$, From (2): $B = \dfrac{15}{\sin\theta}$, $\therefore \dfrac{20}{\cos\theta} = \dfrac{15}{\sin\theta}$, $\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \dfrac{15}{20}$

Hence, $\tan\theta = \dfrac{15}{20} \Rightarrow \theta = 37^\circ$ and $B = \dfrac{15}{\sin\theta} = \dfrac{20}{\cos\theta} = 25$ N (2 s.f.)

Worked Example 2.

The four forces in the diagram are in equilibrium. What are the values of D and θ?



Written by T. Graham, M.C. Harrison, S. Lee, C.L.Robinson

Solution

Resolving horizontally: $13\cos 22° + \ 4\cos 70° - D\cos\theta - 16\cos 60° = 0$

$D\cos\theta = 5.421$ N (1)

Resolving vertically: $13\sin 22° - \ 4\sin 70° + D\sin\theta - 16\sin 60° = 0$

$D\sin\theta = 12.745$ N (2)

An alternative method of solution to equations (1) and (2) to that used in Worked Example 1 makes use of the identity: $\sin^2\theta + \cos^2\theta = 1$, for any θ. Squaring both sides of (1) and (2) gives $D^2\cos^2\theta = 5.421^2$ (3) and $D^2\sin^2\theta = 12.745^2$ (4)
Adding (3) and (4):
$D^2(\cos^2\theta + \sin^2\theta) = 5.421^2 + 12.745^2$, $D^2 = 191.8$, $D = 13.8$ N $= 14$ N (2 s.f.)
Then from (1): $\cos\theta = \dfrac{5.421}{D} \Rightarrow \theta = 67°$

Exercises

1. The three forces in diagram 1 are in equilibrium. What are the values of F and θ?

2. Two light inextensible strings suspend a particle of weight 78N. The angle between each string and the vertical is 42°, as shown in diagram 2. What is the tension in each string?

3. The four forces in diagram 3 are in equilibrium. What are the values of F and θ?

4. The five forces in diagram 4 are in equilibrium. What are the values of F and θ?

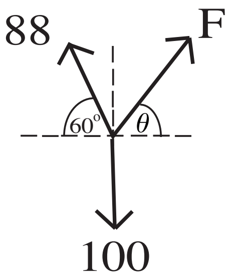


diagram 1

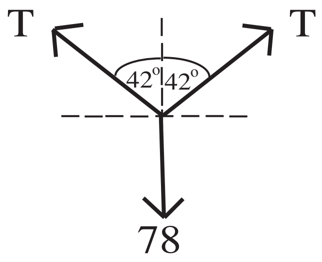


diagram 2

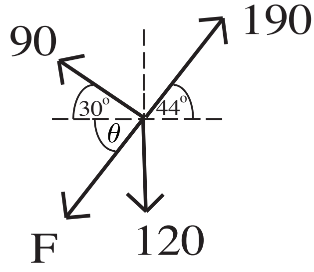


diagram 3

Answers (All 2 s.f.)

1. F = 50 N, $\theta = 28°$
2. Tension in each string = 52 N
3. F = 82 N, $\theta = 44°$
4. F = 200 N, $\theta = 26°$

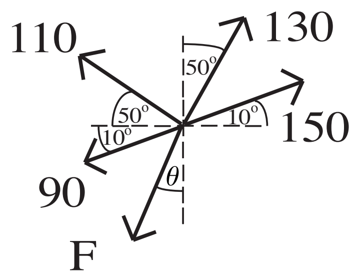


diagram 4

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