

## Ordinary differential equations

To solve  $\frac{dy}{dx} = f(x, y)$  :

### Euler's method:

$$y_{r+1} = y_r + hf(x_r, y_r).$$

### Modified Euler method:

$$y_{r+1}^{(p)} = y_r + hf_r \quad f_{r+1}^{(p)} = f(x_{r+1}, y_{r+1}^{(p)}).$$

$$y_{r+1}^{(c)} = y_r + \frac{h}{2}(f_r + f_{r+1}^{(p)}).$$

### Runge-Kutta method:

$$k_1 = hf(x_r, y_r), \quad k_2 = hf\left(x_r + \frac{h}{2}, y_r + \frac{k_1}{2}\right).$$

$$k_3 = hf\left(x_r + \frac{h}{2}, y_r + \frac{k_2}{2}\right), \quad k_4 = hf(x_r + h, y_r + k_3).$$

$$y_{r+1} = y_r + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$